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Herding in Financial Markets

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Abstract

Our approach in this paper is to analyse investment behaviour, primarily by focusing on their propensity to exhibit herd behaviour. For this, we will use the Nasdaq index. We will be looking at if the proposed index reflects the behaviour of the agents and which dynamic can be explained by the Kirman model. For this, we will create an opinion index and compare with a beta distribution. While assessing our index, the use of statistical methods shows that we can receive an intriguing and useful explanation of various stylised facts of financial markets.

Keywords: Behavioural finance, Stylised facts of financial markets, Herding behaviour, Kirman model, Stock markets.

JEL Classification: C63, D84, G1, G2, G15, F4

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1. Introduction

The aim of this project is to know if we can explain the investment behaviour of the market agents, for this we make use of Kirman's model (1993). From the index proposed by Vidal D. together with Simone A. for the S&P500, we want to see if it is applicable to other financial markets, such as the Nasdaq index, which will be the object of our study. Once we clarify what we want to prove, we need a means and that is achieved through the Beta distribution, as I will mention later, if the opinion index follows this distribution we can say that the dynamics of the index are explained by the Kirman's model. In our case, for the index created based on the Nasdaq, it can be seen that it is quite close to the Beta distribution, so we can say that it seems that the Kirman's model accurately reflects the behaviour of the agents. As a complement, we will also verify if the empirical properties of the assets returns are met, specifically we will be focused on three stylised facts, which will confirm heavy tails, also uncorrelation and volatility cluster in returns distributions.

Following this, due to the fact herding behaviour plays a vital role in our approach we shall continue our study by reviewing other works within this field of research. Herding behaviour can be defined as, *"the phenomenon of individuals deciding to follow others and imitating group behaviours rather than deciding independently and atomistically on the basis of their own, private information"* according to Baddeley (2010). Even Keynes was a pioneer in the field of herding, his study focused on analysing the impulse to copy and following the behaviour of the majority in a doubtfulness world Keynes (1930). For Keynes, herding was an answer to the indecision and personal' insights of people about their particular inexperience: it was assumed that people imitated the behaviour of the crowd since there was an idea that the crowd was better informed and therefore did the right. Such as, a Keynes famous quote (1936) is that: *"Worldly wisdom teaches that it is better to fail conventionally than to succeed unconventionally"*. This process of herding can lead to instability and financial markets generate speculative episodes that lead to bubbles and cracks. However, these considerations by Keynes have been somewhat neglected in economists' explanations of herding.

On the other hand, because herding can not easily be explained as the result of rational expectation theory, economists have tended to retain the assumptions of rationality, but in a weakened form. This lack has been met with different statistical hypotheses - by example, Kirman (1993), using an approach of Markov chain, displayed its model of ants which the ants become copying another ant; for example, the ants that face two symmetrical resource fonts will be disposed to group themselves in one or

another source (instead of distributing itself uniformly between both). This behaviour can be interpreted like activity of recruitment by the ants - when there are beneficial outcomes of the behaviour of search for food, the cooperative exploitation of a font will add more profit to the crowd that a uniform distribution of the effort on two unlike fonts Kirman (1993).

Other academic, tentative and experimental studies, like, for example, Scharfstein and Stein (1990) demonstrate theoretically that could be acceptable for the negotiators to imitate if they worry about his reputation. In addition, Palley (1995) carry out a model in which herd behaviour happens if the agents are averse to the risk and if his bounty is subject to its relative efficiency. During the periods of uncertainty, the agents look for what he calls “*security in numbers*”.

Cipriani and Guarino (2009, 2014) approve empirically and experimentally that the behaviour of rational herding can increase because of the uncertainty. Making an investigation homework to discover the reasons of the collapse of stock market of United States of America in October of 1987, Shiller (1990) concludes that: “*The suggestion which we have on the causes of the crash is the reaction between people with a greater attention and emotion, treating to understand what other investors were prone to do, and falling in intuitive models, like models of price reversal and continuation*”.

On similar way, Shiller and Pond (1989) discover that herd behaviour emerges, not only in stages of crisis, but also, when the prices raise fast. Give the impression that individual and institutional shareholders are more prone to be focus of corruption effects if they suffer anxiety. Additionally, Hommes et al. (2005) and Heemeijer et al. (2009) make experiments of learning-to-forecast, and they inform that the subjects tend to coordinate common strategies of prediction. The behaviour of coordination of the subjects is particularly strong in systems of positive feedback like the financial markets. On the other hand, Chiang and Zheng (2010) notice strong evidences of herding behaviour in the worldwide stock markets. They affirm that herding behaviour is present in the ascending and descending markets and, that is higher in unsettled phases than in the serene ones. Other psychological tests contributed by Prechter and Parker (2007) and Baddeley (2010) reveal that, in situations of confidence, the individuals are disposed to rationalise knowingly, nonetheless they have an instinctively tendency to herd in situations of indecision.

Based on the previously discussed ideas and while focusing on our model, we accept that due to the past volatilities of the market, there is a growth in the correlation between the arbitrary sides of the trade signals of the speculators. That is, from an economic viewpoint, that speculators observe more in detail what others do, in doubtfulness phases. It is shown that this type of herding behaviour, which coincides

with the experimental and tentative research, affects the price dynamics besides the heterogeneity of applying trade rules. Another consideration is that the change of stock values happens randomly. In other words, they are practically impossible to predict. This result occurs because speculators rely on a wide variety of trading rules that vary over time. On the other hand, if speculators coordinate their behaviour, extreme statements can arise, leading to fat-tail return distributions... as we will see it.

We use a set of data of the NASDAQ to estimate our model. This data set, which goes from January 2000 to December 2016, contains 4237 observations, which are distributed daily. The observations correspond to the 71 shares traded on the same days as the index. This data has been downloaded from Yahoo Finance. To verify the section of the stylised facts in financial markets, we proceeded to calculate the returns and follow the steps as indicated in Cont, R. (2001).

Other models with heterogenous interactions of agents can also explain the changes in volume of the financial markets (a general overview, it is seen LeBaron 2006, Chiarella, Dieci and others, 2009, Hommes and Wagener 2009, Lux 2009). In Day and Huang (1990), the speculators are focused on nonlinear commercial rules. In Brock and Hommes (1998), the speculators change between the methodological and essential regulations of commerce in function of the passed yield of regulations. Bouchaud and Cont (1998) display an approach of Langevin to describe the underlying forces of the height and the bankruptcy of the stock markets. The artificial models of the stock market of LeBaron and others (1999) and Chen and Yeh (2001) produce accurate dynamics because of the connections of several diverse kinds of speculators. Kirman (1991) and Alfarano and Lux (2007) suggest herding models in which speculators can persuade other speculators to imitate his behaviour. In Lux (1995) and Lux and Marchesi (1999), the behaviour of selection of rules of the speculators is predisposed by the characteristic appreciation of the rules, between other issues, so consequently a rule can increase his appreciation if it has several imitators. In Cont and Bouchaud (2000) and Stauffer and others (1999), the herding behaviour of the opportunists is of local ambit. In these models, the speculators are located in a network, but all the sites of the network are not occupied. The speculators who form a local neighbourhood, that is to say, a group of occupied sites connected, buy or sell assets collectively. In the herding models of Iori (1999, 2002), the speculators can buy or sell a unit of risk assets or remaining inactive. This decision is ingrained by the public interaction, i.e. by the opinions of speculators' neighbours. Additionally, the commercial frictions imply that the changes of previous prices influence in the level of activity of the speculators. On the other hand, Tedeschi et al. (2012) develop a herding model in which speculators imitate the behaviour of the

most successful speculators and, peculiarly, show that the speculators have a propensity to imitate and a desire of being imitated, since the herding is beneficial.

Observe that in Kirman (1991), Lux (1995), Lux and Marchesi (1999) and Alfarano and Lux (2007), the behaviour of the speculators influences in if they decide on a methodological or fundamental rule, in addition, along with Cont and Bouchaud (2000) and Stauffer et al. (1999), the behaviour of the speculators in the herding determines if they are optimistic or pessimistic. Since the speculators take into account the actions from other speculators, the heterogeneity of the applied rules of trading becomes variable in the time. Of the existing models of herding, the model of Kirman (1991) is the one that is more closely related to our model of herding. In spite of some differences, between the described models and the one that we used, they have in common that the social interactions and the conduct of imitation depend on the dynamics of previous prices of the market.

The rest of our work is organised as follows. In section 2, we deploy the set of data that we will use for our project, also comment a brief history about the index that we are going to analyse and then check the stylised statistical properties of Nasdaq' returns, to finish this part, we proceed to the construction of the empirical opinion index. Next, in section 3, we do an investigation on the previous empirical research about this subject and all followed it is explained the herding mechanism. Further on, in section 4, we show the empirical results for the Nasdaq Composite index. Finally, section 5 concludes our project and sets a series of guidelines for possible future research.

2. Data

2.1 Nasdaq Composite

NASDAQ Composite (^IXIC) is a stock-exchange index of the common stocks and similar securities (for example, ADRs, tracking stocks, interests of limited society) that quote in the stock market of NASDAQ. Together with the Dow Jones Average and S&P500, Nasdaq is one of the three most followed indices in the U.S. stock market. The composition of NASDAQ Composite is strongly oriented towards the companies of technology of the information.

Figure. 1: NASDAQ Composite (^IXIC) from 17/06/1977 to 09/06/2017. Source: Google Finance



After coming to the light in 1971 with 50 companies and an initial value of 100, the 10 of March of 2000, NASDAQ Composite reached a maximum of 5132.52 (and a closing price of 5048.62) during the first bubble of Internet (1), and later, the 10 of October of 2002, fell to a minimum of 1108.49 when the bubble exploded (2). The high prices of the energy and the possibility of recession reduced NASDAQ to a bearish market at the beginning of 2008, that was recognized the 6 of February of that same year when NASDAQ closed below the level of 2,300, near a 20% below the recent maximums (3). From there, it has followed quite positive a trajectory ascending that lasts to the present time.

As mentioned earlier, the data used both to verify the empirical properties of asset returns and to create the opinion index are extracted from Yahoo Finance. It is a daily sample that covers from 03/01/2000 to 12/30/2016 and is composed by 71 assets that have quoted the same days of the index (neither have broken nor have they had

temporary suspension). The sample of the Nasdaq Composite was chosen since there has not been a previous study done, a difference with the S&P 500, which has been used by David Vidal-Tomas and Simone Alfarano in his research on ‘Herding behaviour in Stock markets’.

2.2 Stylised statistical properties of Nasdaq’ returns

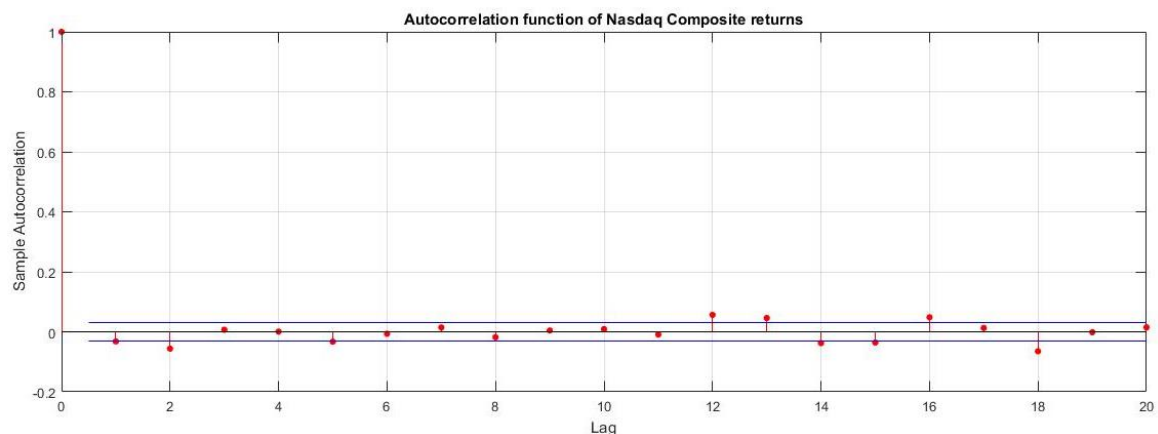
Cont, R. (2001) presents an amount of stylised empirical facts that arise from the statistical study of price changes in many kinds of financial markets. To begin to work with this paper, it is necessary to clarify what stylised facts are, which we can define as properties common to an extensive variety of tools, markets and phases. They are attained by selecting a joint denominator between the characteristics detected in reviews of diverse markets and tools. Obviously, in doing so, it is gained in generally, but it has a tendency to diminish in the accuracy of the declarations that can be made on the yields of the assets. Once this is clarified, we proceed to calculate the returns that will serve us for the analysis of the properties that we want to contrast. So, given the time series of prices, in our case (^IXIC), from the adjusted prices at closing (AdjClose), we calculate the returns $r(t)$:

$$r(t) = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

We will start by checking:

1. Absence of autocorrelations: Nasdaq returns are uncorrelated, what means that there aren’t repeating patterns. Therefore, is difficult to predict the future through the past returns.

Figure. 2: Autocorrelation function of Nasdaq Composite returns



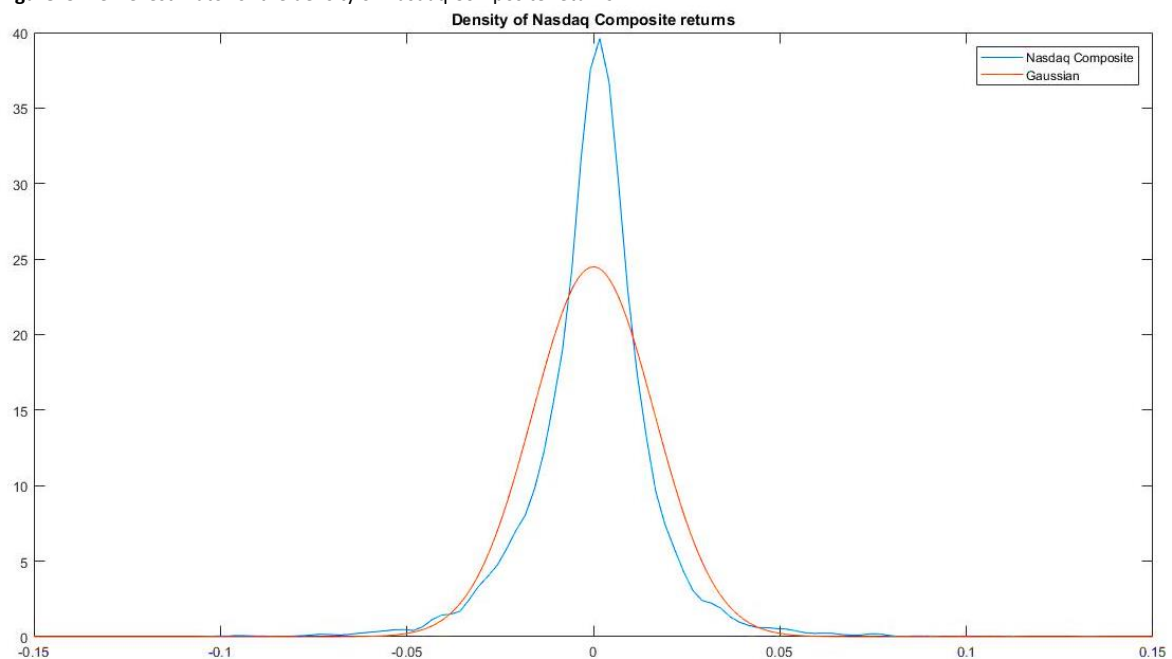
The autocorrelation function of the returns:

$$C(\tau) = \text{corr}(r(t, \Delta t), r(t + \tau, \Delta t))$$

(where corr represents the sample correlation) quickly decays to zero in a few lags: for $\tau \geq 19$, it can be safely supposed to be zero for all practical purposes.

2. Heavy tails: It can be observed when comparing the empirical distribution with a Gaussian distribution. The tail of the first surpasses the second when they approach the ends.

Figure. 3: Kernel estimator of the density of Nasdaq Composite returns



It can be seen how our series does not follow a normal distribution, nevertheless to make sure we realize the Jarque-Bera test with the hypotheses:

- $h = 0$; The series follows a standard
- $h = 1$; Rejects the null hypothesis of normality

The results are as follows: $h = 1$; $p = 1.0000e-03$ with which we can affirm that we reject normality at the 1% significance level.

We also get the value of Skewness that is “a method of quantify the asymmetry of the statistics about the sample mean. If skewness is negative, the data are extended more to the left of the mean than to the right, while if skewness is positive, the data are extended more to the right. The skewness of the normal distribution (or any perfectly

symmetric distribution) is zero.” In our case, we obtain a value of 0.0320 which indicates us that it is quite symmetrical, a fact that we could appreciate from the previous graph.

To carry out the calculation, the following formula is used:”

$$s = \frac{E(x - \mu)^3}{\sigma^3}$$

where μ is the mean of x , σ is the normal deviation of x , and $E(t)$ characterises the predictable rate of the amount t .”

Finally, we calculate the Kurtosis which is “a method of quantify how outlier-prone a distribution is. The kurtosis of the standard distribution is 3. Distributions that are more outlier-prone than the standard distribution have kurtosis larger than 3, these are those that have heavy tails, because their distribution is concentrated in a location and therefore, explodes by the lateral ones; and distributions that are less outlier-prone have kurtosis less than 3.” The value we get is $8.6987 > 3$ and we can add that it is a *Leptokurtic* distribution.

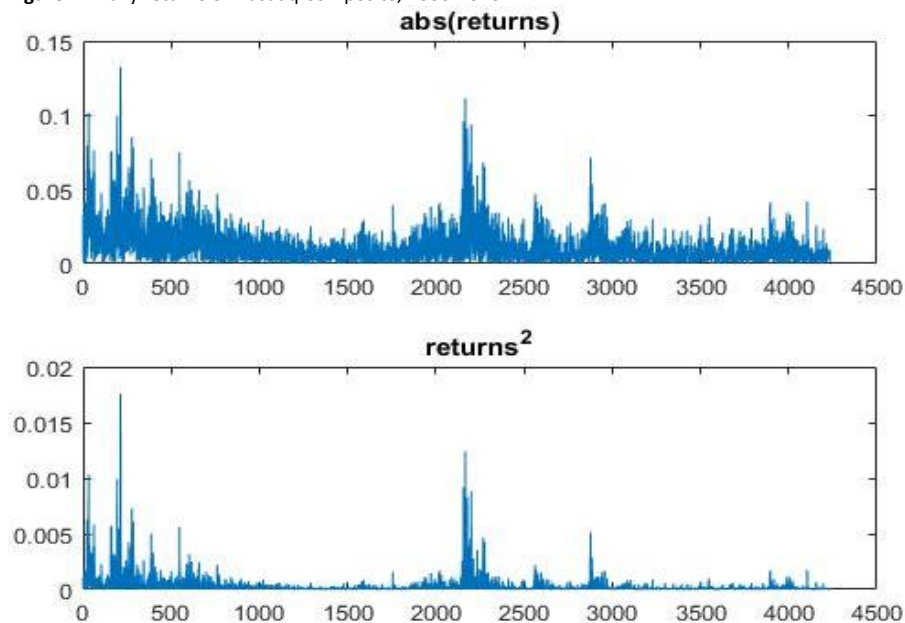
“The kurtosis of a distribution can be defined as:

$$k = \frac{E(x - \mu)^4}{\sigma^4}$$

where μ is the mean of x , σ is the normal deviation of x , and $E(t)$ characterises the predictable rate of the amount t .”

3. Volatility clustering: various contrasts of volatility show a favourable autocorrelation during some days, which demonstrate the point that high-volatility events have a tendency to cluster in time.

Figure. 4: Daily returns of Nasdaq Composite, 2000-2016.



We provide two measures of volatility: *abs (returns)*, absolute value of returns and *returns*², the square of returns. With both we can reach the same conclusion and is that the largest volatility clustering occurred during the internet bubble and followed by the financial crisis.

2.3 Empirical opinion index

To calculate this index, we are based on the previously developed by David Vidal-Tomas and Simone Alfarano in his research on 'Herding behavior in Stock markets'. In summary, they compute n as the amount of singular positions ($n_{i,t}$) of every stock in relation to its exponential moving average (EMA) in period t . The pessimistic status for every stock is $n_{i,t} = 1$ when the prices are lowering below 40-days EMA at moment t , whereas, the optimistic status is characterised by $n_{i,t} = 0$ when the prices raise more than 40-days EMA in the time t .¹

$$\begin{cases} n_{i,t} = 1 \text{ if } price < 40 - \text{days EMA} \\ n_{i,t} = 0 \text{ if } price \geq 40 - \text{days EMA} \end{cases}$$

Consequently, we can perceive the status of the marketplace (pessimistic/optimistic) determined by the amount of all statuses ($n_{i,t}$) at period t , assumed N stocks. In this background, " z_t^e " is an empirical opinion index which characterises the common behaviour (status) of all stocks in the period t ($\sum_{i=1}^N n_{i,t}$) related to the stock market as an entire (N).²

$$z_t^e = \frac{n_t}{N} = \frac{\sum_{i=1}^N n_{i,t}}{N}$$

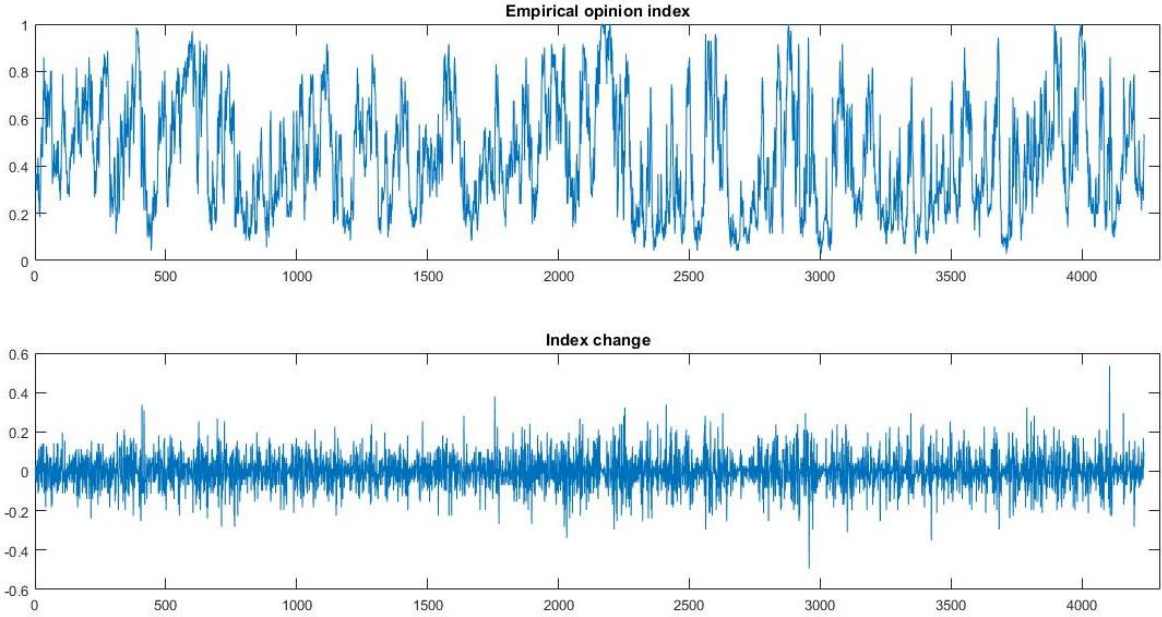
Finally, we provide a version of the empirical opinion index we have calculated.³

¹The exogenous election of 40 days occurs by the necessity to have a time period neither too long nor short.

²When " $z = 1$ " we perceive a bear market, whereas, when $z = 0$, we are in a bull market".

³The diverse kind of calculations and graphs have been elaborated by means of using MATLAB. On the x-axis, the values are distributed by observation number related to the analysed period from 03/01/2000 to 12/30/2016.

Figure. 5: Empirical opinion index of Nasdaq Composite, 2000-2016 and its respective variation.



3. Previous Empirical Research

3.1 Models of interaction in financial markets

As mentioned in Lux T. & Alfarano S. (2016) for analyse these models of interaction, we need to forget the idea of the rational bubble theory. Since, the universal cubic law of price returns, the perspective of the power law and the long-term correlations of volatility suggest to consider the financial data like the results of a social process of interactive agents. Therefore, models based on rational expectations, including the theory of the rational bubble, do not contain anything of that.

The early formal models of interactive groups of traders can be found in Baumol (1957) and Zeeman (1974). These authors differentiate between two types of traders: first, the fundamentalists, consider that the prices of the assets are determined by fundamental factors only. These traders would sell/buy if they suppose that the present price of the market is underneath/over the rationally calculated fundamental value. The second group, in its majority called chartists or noise traders, prefers to be convinced that the markets of assets are impelled by regular tendencies and that exist patterns that could be extracted via moving averages, regressions or more complex measures. Hence, the price of market is from the interaction of both groups and their supply and demand.

First multi-agent model in which appears evidence of volatility clustering as an emergent attribute can be find in Grannan and Swindle (1994). It took a time so that the explanations of the empirical characteristics became a subject in this chain of literature (LeBaron et al. (1999), Chen and Yeh (2002), Lux and Schornstein (2005)). Normally, it is observed certain tendency towards the heavy tails and volatility cluster, though the numerical results are often far from empirical power laws. Next, it is review with more detail models of interaction of the financial markets that cover great part of the available literature until now.

An initial view in this path can be found in Kirman (1991), (1993) who adapts a basic random model for the transmission of information in colonies of ants to model changes of strategies of traders in financial markets. The initial setup has ants switching information about the way of resource fonts, the adaptation to a financial situation replaces by foreign exchange dealers switching information about the exactitude of the chartists and fundamentalist predictions of the types of change. The agents who face other traders adopt their strategy with certain probability, but the agents also can experience independent changes of opinion without interaction.

In the other hand, Alfarano and Lux (2007) have a reduced version of a herding model which still seems to make the work of generate the appropriate power laws for the returns and volatility. Again, adopt that two different groups interact in the market, fundamentalist and noise traders who follow the present disposition of the market. Whereas the first group negotiates simply on the base of the observed price of appraisal (that is to say, the differences between the price p and the fundamental value p_f), noise traders are assumed that they are influenced by the dynamics of contagion. There are two options, pessimistic (sellers) or optimistic (purchasers) and to change among either status of the market with simple probabilities revealing the impact of the crowd decision:

$$prob(O \rightarrow P) = v \frac{N_p}{N}, \quad prob(P \rightarrow O) = v \frac{N_o}{N} \quad (1)$$

with “ $N_p(N_o)$ the number of pessimistic (optimistic) agents, v a parameter of time scale and $N = N_o + N_p$.” Adding the excessive demand as much of fundamentalists as of noise traders, the general difference between the demand and the supply can be written like:

$$ED = T_F(p_f - p) + T_C x, \quad x = \frac{N_o - N_p}{N} \quad (2)$$

with T_F , T_C constants determining the trading volume of fundamentalists and noise traders. Supposing that the stability of the market is reached instantaneously, the price of equilibrium can be solved:

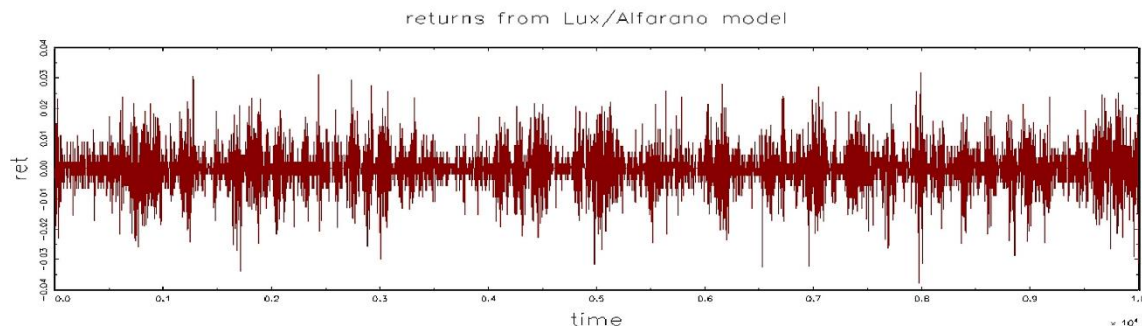
$$p = p_f + \frac{T_C}{T_F} x. \quad (3)$$

As it is possible to be observed in (3), the changes of prices are generated as much by exogenous entrance of new information about fundamentals (p_f) as endogenous changes in supply and demand produced through herding mechanism, whereas traditional financial models only allow the first component (for $x = 0$) and then, we must track all the characteristics of the returns with similar characteristics of fundamentals. Behavioural finance models give a paper to the intrinsic dynamics of the financial markets. Although this structure is alike to the model of rational bubble, contrary to this, the second aspect of the equation (3) doesn't have to follow a limitation for the rationality or uniformity of the expectations.

As demonstrated in a typical simulation in Fig. 6, these few elements described before are enough to produce accurate temporal series for returns whose distribution and temporary features are very nearby to observed results. The herding mechanism of Eq. (1) produces a bimodal restrictive distribution for the section of noise traders in the two groups of pessimistic and optimistic traders. In most cases, one, hence, meets with

most of the noise traders on the supply/demand side of the market (following the undervaluation or overvaluation of the asset price). Nevertheless, the stochastic nature of the process also leads to recurring changes from one crowd to another. Through these phases, the large oscillations in the average opinion lead to an increase in volatility that will last for some time until a blockage happens again to a steady optimistic or crowd pessimistic.

Figure. 6: The conducting influence are the stochastic transitions of agents among trading strategies which lead to exchanges between episodes of high and low volatility. Source: T. Lux, S. Alfarano/Chaos, Solitons and Fractals 88 (2016) 3-18



As pointed in Alfarano and Lux (2007), at least the time scale of volatility in this model does not follow a true power-law and because of the Markovian structure of the model, there can not be 'true' long-run dependency. One more extension of the simple herding dynamics displayed previously, is the early model of Lux and Marchesi (1999), (2000) who adds transitions among noise traders and fundamentalists based on the profit of each strategy. Lux and Marchesi demonstrate that this configuration, relatively complex, could reproduce realistic empirically laws, for both the returns as for volatility. Several analyses of sensitivity also show that the numerical results are not very sensible with respect to parameters variations. The representative changes among calm and volatile phases are caused by recurrent temporary deviations of an equilibrium, in which the price is near its fundamental value. The mechanism is the following one: in the neighbourhood of the stability neither erroneous prices, nor any predictable patterns in the paths of prices exist, so that neither chartist, nor the fundamentalist strategy has an advantage. The agents, therefore, alternate among these alternatives of a little systematic way that does that the composition of the population (in terms of strategies) follows a random walk. Nevertheless, the stability of the fundamental equilibrium mainly depends on the proportion of chartists and fundamentalists.

As it is possible to be observed, the deterministic approach of the dynamic system displays that one can interpret the sum of chartists and fundamentalists like a critical parameter of a simpler system in which the changes of this variable lead to movements back and forth through a Hopf bifurcation scenario. The recent results on some simpler

versions Alfarano et al. (2005) suggest, in some circumstances, the interactive models of this type can generate true power laws for returns and volatility. Lux and Marchesi (2000) maintain that independently of the specific details of the model, the indeterminateness of the composition of the population in a market equilibrium could be a relatively general phenomenon (due to the absence of profit of any commercial strategy in a stability situation) and, along with the dependency of the stability in the composition of the population, would have to exist a possible intermittency in a wide class of behavioural finance models.

However, it is necessary to mention that there is an important deficiency in these models: its result usually depends on sensible way of the size of the system (that is to say, the number of agents that operates in the market). With the growth of the size of the population, the dynamic characteristics and the statistics of power-law get lost (Egenter et al. (1999)). The reason is that, with an increasing number of independent agents, a law of great numbers enters game and stochastic dynamics becomes the equivalent one to the form of a Normal distribution. Nevertheless, Alfarano et al. (2008) and Irle et al. (2011) show that the size dependency could be a consequence of the exact topology of the interaction of the agents. Specifically, to assume a given intensity of interaction of the agents with its neighbours implies that with the variation of the number of agents (the size of the system), the relative force of this component will diminish. To scale the component of the interaction of a parallel way to the increase of the size of the system will conserve interesting dynamics and will make qualitative outcome independent from the number of agents. The increase of the speed transmission of the information with modern means of communication could provide a justification that supports an increase of the range of interactions. Alfarano and Milakovi (2004) have proposed an alternative solution to the dependency of the size of the system in the models of herding when introducing particular hierarchical structure between the participants in the market.

3.2 Herding Mechanism

From Alfarano, S., Lux, T., & Wagner, F. (2005) the dynamic forces of the market are defined as “a jump Markov process in continuous time”. The market is occupied by a static number of negotiators N , every of them being between the status 1 or 2. The quantity of negotiators in the first status is symbolised by n , therefore $N - n$ is for the second status. The conditional probabilities $\bar{p}(n', t + \Delta t / n, t)$ of moving from n at time t to n' at time $t + \Delta t$ with $n' - n = \pm 1, 0$ through a unique change are linked to the transition

probabilities $\pi(n \rightarrow n')$ per unit time by $\bar{\rho}(n', t + \Delta t / n, t) = \Delta t \pi(n \rightarrow n')$. The last are provided such as:

$$\pi(n \rightarrow n+1) = (N-n)(a_1 + b n) \quad \text{and} \quad \pi(n \rightarrow n-1) = n[a_2 + b(N-n)].$$

[Adds pessimist][71][Herding parameter][Group 1][subtracts pessimist][Group 2][Random events]

The constants a_1 and a_2 define the distinctive tendency to modify the status, whereas the term b captures the propensity to herd. Opposite to previous ideas, it is allowed that the two constant parameters, a_1 and a_2 , to adopt diverse estimations, producing, therefore, the necessary asymmetric behaviour. The transition probability $\pi(n \rightarrow n)$ to continue in the same status results from the condition $\sum_n \bar{\rho}(n', t + \Delta t / n, t) = 1$. Due to transition probabilities should continue limited to values < 1 per period increase, discrete reproductions of the model would only be conceivable until a superior limit of Δt which is granted, for big N , by $\Delta t = (2 / bN^2)$. [...] After a long process of calculations it arrives at the equilibrium distribution:

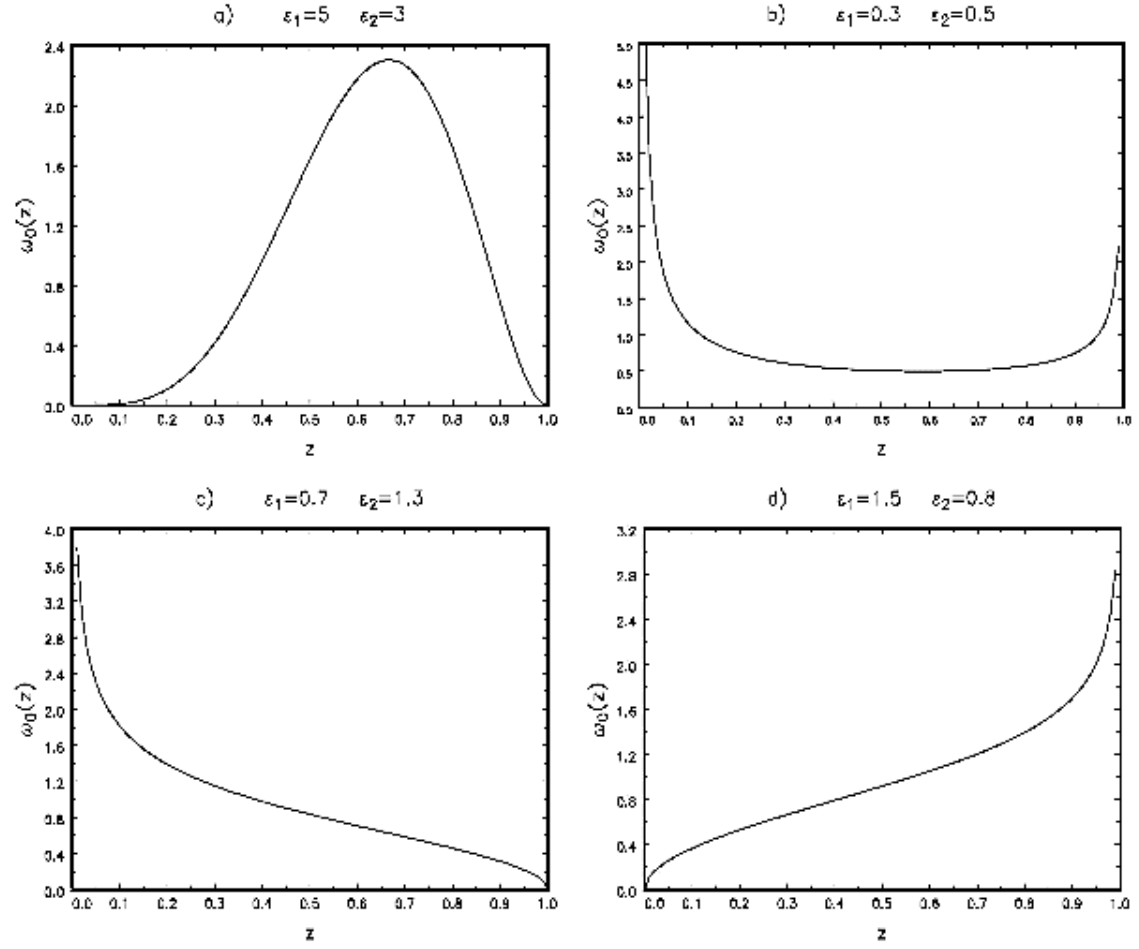
$$p_e(z) = \frac{1}{B(\varepsilon_1, \varepsilon_2)} z^{\varepsilon_1 - 1} (1 - z)^{\varepsilon_2 - 1} \quad (4)^*$$

$$\text{where} \quad B(\varepsilon_1, \varepsilon_2) = \frac{\Gamma(\varepsilon_1) \Gamma(\varepsilon_2)}{\Gamma(\varepsilon_1 + \varepsilon_2)} \quad (5)$$

The distribution (4) is known in the probability literature as the Beta distribution, which is one of the most flexible distributions in a bounded domain. Figure 7 shows different types of equilibrium distribution functions embedded in the formula (4). It is a remarkable property of the model that it possesses such extreme flexibility of the resulting equilibrium distribution, despite the very few underlying parameters.

* $\varepsilon_1 = \frac{a_1}{b}$, $\varepsilon_2 = \frac{a_2}{b}$

Figure. 7: The four images display diverse probability densities derived from Equation (4) for different selections of the parameters ε_1 and ε_2 . In the image (a), an uni-modal distribution, in (b), a bi-modal distribution and in image (c) and (d), two cases of asymmetric monotonic distributions are shown.

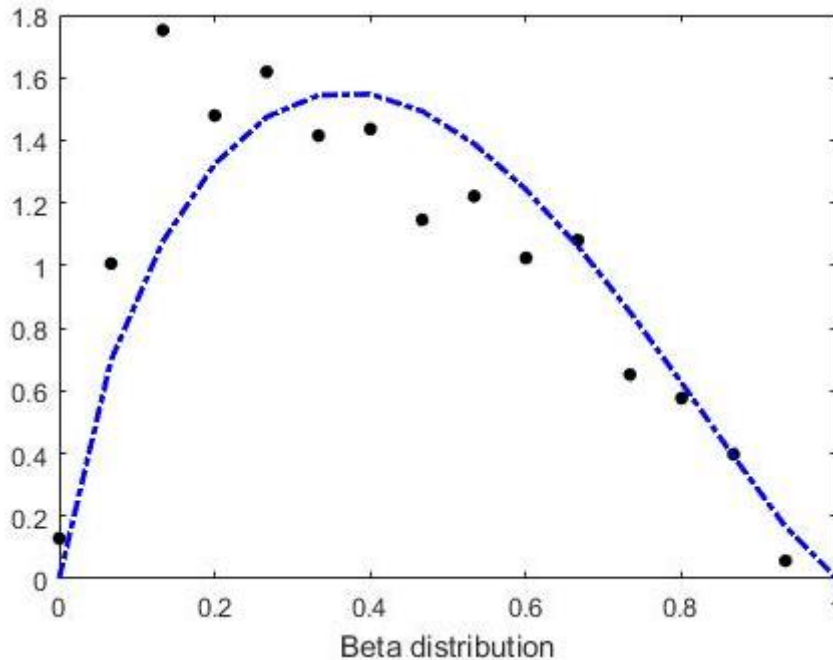


4. Empirical Results

Once we clear the way to understand the Herding mechanism, we have to transfer the theoretical demonstration to the program we use (Matlab), from there, after we have prepared our empirical opinion index, proceed to the estimation of the parameters of the Beta and that, we carry out by means of the '*betafit*' command that returns the most successful value for each parameter (ε_1 and ε_2) as well as the limits that border on the top and bottom. As soon as this has been obtained along with the interval to be considered taking into account the maximum value, minimum, and the number of intervals defined, we create the theoretical Beta, which serves as a basis for comparing it with the real data and thus appreciate that follows such distribution.

Afterward, we plotted the Beta distribution with our data on the Nasdaq Composite. We obtain Figure 8 on which we can see how the density of probability moves to less pessimism, in addition, we could also add that follows a uni-modal distribution with values $\varepsilon_1 = 1.7668$ and $\varepsilon_2 = 2.3065$. Finally, since the Beta approaches the empirical distribution we can say that the z index is explained by the Kirman model.

Figure. 8: This figure displays the probability density function of the empirical opinion index.



5. Conclusion

We have begun the present work doing a route through the main theoretical contributions on herding behaviour, to go to analyse later the difficulty to measure the herding effect in the financial context. We have continued with the demonstration of some stylised facts and, finally, we have constructed an empirical opinion index, which we have compared it with a Beta distribution to reach the conclusion that the Kirman model can serve to us to gather the herd behaviour.

In behavioural finance literature, has been proposed several types of models with interaction of diverse associations of traders. The conclusions of the previous researches are that we can exclude the rational part from the models of speculative activity. And also, it is possible to be appreciated as the speculators observe with more interest the actions of other speculators in periods of greater uncertainty, which leads to a diminution in the heterogeneity.

Our empirical analysis is focused on the Nasdaq Composite. We have had the luck to have access to 16 years of daily data to make our studies. With it, we have tried to display with certain details a statistical data set that arises from the empirical study of the asset returns and that are common to a great set of assets and markets. We show that our index reproduces some of the most important emergent properties in the time series of returns: absence of autocorrelations, fat tails and volatility cluster.

In addition, in this work, we have elaborated an opinion index that is capable of show the behaviour of the agents as a result of the oscillations of the price of the shares. Inspired by the work made by Vidal-Take D. and Alfarano S., the opinion index oscillates in sync with the location of the prices relative to its own EMA of 40 days between $z_d^e = 1$, extreme pessimistic and $z_d^e = 0$, extreme optimistic. Finally, one demonstrates that z_d^e can suitably be explained by the Beta distribution, and, therefore, that z_d^e can be explained generally by the Kirman model.

The Kirman model can not represent many negative cases perfectly, that is to say, exogenous changes that cause that all prices of the actions diminish simultaneously for a long period of time. Nevertheless, despite that, Kirman model represents one first approach that can describe 16 years of stock market, with their bubbles and crisis, just by three constant parameters: a_1 , a_2 and b .

As final note, we believe that our results help to this field in two ways. In the first place, the point that the model of Kirman is able to explain in general the empirical opinion index, z_d^e , grants us to know the restrictions that cause the inside dynamic forces of the stock markets, in this specific occasion, the Nasdaq Composite index. And second,

the use of z_d^e allows us to represent the behavior of million traders through a limited number of shares. Alternatively, future revisions can be done to predict the pessimism and the optimism of the agents through dispersion method (Langevin Equation) or taking into account other distributions.

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